MULTI-CRITERIA GROUP DECISION MAKING METHOD BASED ON FUZZY SETS APPROACH FOR SUPPLIER SELECTION PROBLEM

Keivan Shahgholian¹; Alireza Shahraki²; Zohreh Vaezi³

These days, the management of supply chain has attracted the attention of most companies since this ensures their survival in a competitive market condition and they can reach to customer satisfaction objects through applying this management system. Competition between companies is no longer meaningful. Instead, competition between supply chains has been evolved. For this reasons, supplier selection problem, plays a significant role as a strategic feature in company's success. In this paper we propose a multi-criteria group decision making approach based on Fuzzy sets which can solve supplier selection problems that have much vagueness. In this method, at first linguistic variables are used to assess the weights for each decision maker. Then the rating and weights of suppliers and criteria are assessed by use of linguistic variables. Finally, suppliers are ranked in terms of their scores in each criterion and the top suppliers are selected. In order to clarify this approach we will describe solution of a numerical problem at the end of this paper.

Field of Research: Supply Chain, Supplier Selection, Fuzzy sets, Multi-criteria decision making

1. Introduction

All activities relating to product flow and conversion of materials, from supplying row materials to presenting final product to a customer, and also related information and financial flows lie inside supply chain. The management of supply chain means to integrate these activities through improving of chain relationships in order to obtain competitive advantages [20]. Goffin et al. (1997) have stated that management of suppliers is a key issues of supply chain management because the cost of row materials and component parts constitutes the main cost of a product and most of the firms have to spend considerable amount of their sales revenues on purchasing.

¹ Department of Industrial Engineering, Faculty of Engineering, Islamic Azad University, Zahedan Branch, Iran
² Department of Industrial Engineering, Faculty of Engineering, University of Sistan & Balouchestan, Iran
³ Department of Industrial Engineering, Islamic Azad University, Zahedan Branch, Iran
Hence, supplier selection is one of the most important decision making problems, since selecting the right suppliers leads to significant saves especially in those companies which spend most part of their sale's income for purchasing raw materials [18]. In average, 70 to 80 percent of value of a product is related to raw material purchasing costs and payments to service providers [11][27]. Therefore, supplier selection is considered as one of the most important problems. Its object is to reduce purchase risk, maximize the total value to the purchaser, and make the closeness and long term relationships between buyers and suppliers [8]. Since human perception and judgment which have ambiguous natures govern decision making process, using Fuzzy sets instead of exact numerical (crisp) will be a suitable selection in these cases. In this paper, we solve supplier selection problem through developing a multi-criteria group decision making approach based on Fuzzy sets. For this, we form a decision maker group and assign a weight to each member within this group. We use linguistic variables in order to rank criteria and define the scores of suppliers in each criterion. The structure of this paper is as follow.

In section 2 we describe the background of this research. In section 3, we introduce Fuzzy sets theory. Some Fuzzy criteria will be defined in section 4 and in section 5 the research method will be described. In section 6, we analyze the proposed method through solving a numerical problem. Finally, this paper ends by conclusion that presented in section 7.

2. literature review

Multi-criteria decision making approaches, propose dozens of methods for supplier selection problems including analytic hierarchy process (AHP), analytic network process (ANP), data envelopment analysis (DEA), fuzzy sets theory (FST), genetic algorithm (GA), mathematical programming, simple multi-attribute rating technique (SMART), and their hybrids [14]. Among various papers which are available for supplier selection problem, whether individual based approaches or combined based approaches, we assessed just those ones which were based on Fuzzy sets approaches.

Sarkar and Mohapatra (2006), used two factors for supplier assessment purposes: a) functionality and b) capability. Their object was to reduce the number of suppliers. Due to this fact that supplier's attributes are ambiguous factors, they used fuzzy sets in order to rank suppliers. Chen et al. (2006) proposed a hierarchy model based on fuzzy sets for supplier selection problem. They used linguistic variables in order to define the weight and rate of elements which could be in the form of triangular fuzzy numbers or trapezoidal fuzzy numbers. They used TOPSIS method for rating the obtained options. Flores - Lopez (2007) proposed a multiple fuzzy model for defining the capability of suppliers in creating customer value. Among 84 factors, which were based the questionnaire response from purchasing managers in US. they used 14

According to our studies there are less methods using heterogeneous group decision making approach. In this approach a weight is assigned to each decision maker which differs with others. In this paper, at first we assign a weight to each decision maker. Then we rate suppliers based on their scores in each criterion using multi-criteria fuzzy decision making approach and finally select the best of them.

3. Fuzzy Sets Theory

For the first time, Professor Zadeh introduced fuzzy sets in the form of an article in the information and control magazine. In this paper he gave fuzzy name to sets had been previously known by Bertrand Russell, John Lucasie Wich, Max Black and the others as ambiguous or multi valued sets [3]. He believed that we need another kind of mathematic in order to empower ourselves to model ambiguities and uncertainties of events [23]. Thus, FST is employed to express the existence of uncertainty in our accurate or mental definitions about preferences, constraints, and goals [28]. This theory can mathematically formulate most non accurate and ambiguous concepts, variables and systems, such as the events of real world, and prepare a context for reasoning, deduction, control and decision making in uncertainty conditions. It should be noted that in this work uncertainty refers to the uncertainty of thoughts and words of human being and differs with the uncertainty of probability theory [19].

3.1. Fuzzy set
A fuzzy set is a set of things in which there is no clear or pre-defined boarder between things which are or are not members of this set. Each thing to some extent may be or may not be a member of this set. Indeed, each member of this set is linked to a value expressing the degree of membership of that member. This value varies between [0,1] in which 0 and 1 stand for the minimum and maximum value of the degree of membership respectively. All other values stand for relative degree of membership [4].

### 3.2. Membership function

A membership function is a function which assigns to each element $x$ of $X$ a number, $\mu_{\tilde{A}}(x)$, in the closed unit interval $[0, 1]$ that characterizes the degree of membership of $x$ in $\tilde{A}$. The closer the value of $\mu_{\tilde{A}}(x)$ is to one, the greater the membership of $x$ in $\tilde{A}$. Thus, a fuzzy set $\tilde{A}$ can be defined precisely by associating with each element $x$, a number between 0 and 1, which represents its grade of membership in $\tilde{A}$.

### 3.3. Triangular fuzzy number

If a triangular fuzzy number (TFN) $\tilde{A}$ be defined as $(a, b, c)$, its membership function is defined as the following [16]:

$$
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{x-a}{b-a} & ; a \leq x \leq b \\
\frac{c-x}{c-b} & ; b \leq x \leq c \\
0 & \text{otherwise}
\end{cases}
$$

In addition, the primary operations for two TFNs are shown in the Appendix A.

### 3.4. Defuzzification

Fuzzy numbers must be transformed into crisp real numbers to obtain a ranking order of alternatives. There are many method have been developed for this purposes. this study adopts the signed distance method among defuzzification methods because of its simplicity and widespread use. The defuzzification of a TFN $\tilde{A}$, by signed distance method, denoted as $d(\tilde{A})$, is therefore given by [6] [24]

$$
d(\tilde{A}) = \frac{1}{4}(a+2b+c) \tag{1}
$$

### 3.5. Linguistic variables

Sometimes it becomes a very difficult task to assess the characteristics of some events through numerical formats. A useful tool which is employed for this purpose is linguistic variables. They are variables which their values are sentences or words of natural or artificial languages [5]. Table 1 presents criteria importance weights and alternative ratings considered as linguistic variables. Fig. 1 shows the respective linguistic variables membership functions of importance weights [8].
4. Fuzzy multi-criteria group decision making

Decision making is a problem solving process by which a method is selected among various methods in order to obtain an effective and applicable result [5]. In real world, we deal with decision making cases which have different, antonym and multiple criteria. If we consider multiple qualitative and antonym elements in our decision making process, we call this a multi-criteria decision making [3]. Multi-criteria decision making have two models: a) multiple objective decisions making (MODM) and b) multiple attribute decision making (MADM). The first model is applied for design purposes whereas the latter is used for selecting top options [2]. As nowadays systems benefit from expert employees in one hand and on the other hand managers of these systems are in the same level, so it would be better to make decisions with respect to the ideas of whole group and the basic body of system's decision makers. We call this kind of decision making: group decision making which can be applied to multi-criteria conditions [21]. Decision making is a complex and difficult process due to various uncertainties and vagueness of information, mentalities and linguistics. So, when we deal with uncertainty conditions in various concepts and processes, we merge fuzzy sets with multi-criteria decision making.
[25]. In multi-criteria decision making method, the weight of elements and the estimated values are expressed by fuzzy numbers or linguistic variables.

5. research method

Step 1: forming decision maker group and defining the priority of decision maker’s weight.

Assume that there is a committee of \( t \) decision makers (DMs), \( D_k, k = 1,2,\ldots,t \). Let \( l_k \) be the importance weight given to individual DMs.

Step 2: Each decision maker defines the importance weights of criteria and sub-criteria.

Assume that there is \( m \) numbers of criteria and \( z \) numbers of sub criteria shown by \( n = 1,2,\ldots,m \), \( y = 1,2,\ldots,z \). Let \( \tilde{W}_{nyk} = (a_{nyk}, b_{nyk}, c_{nyk}) \), be the linguistic weight given to sub-criteria \( C_{ny} \), by DM \( D_k \). The aggregated fuzzy sub-criterion weight with respect to its criterion \( C_n \), denoted as \( \tilde{W}_{ny} = (a_{ny}, b_{ny}, c_{ny}) \), assessed by \( t \) DMs is defined as:

\[
\tilde{W}_{ny} = \sum_{k=1}^{t}(l_k \otimes \tilde{W}_{nyk})/\sum_{k=1}^{t} l_k, n=1,2,\ldots,m, \ y = 1,2,\ldots,z. \tag{2}
\]

The defuzzification of \( \tilde{W}_{ny} \), denoted as \( d(\tilde{W}_{ny}) \), can be obtained by Eq. (1). The crisp value of normalized weight for sub-criterion \( C_{ny} \), denoted as \( W_{ny} \), is given by

\[
W_{ny} = \frac{d(\tilde{W}_{ny})}{\sum_{y=1}^{z} d(\tilde{W}_{ny})}, n=1,2,\ldots,m, \ y = 1,2,\ldots,z, \tag{3}
\]

where \( \sum_{y=1}^{z} W_{ny} = 1. \)

Let \( \tilde{W}_{nk} = (a_{nk}, b_{nk}, c_{nk}) \), be the linguistic importance weight given to criteria \( C_n \), by DMs. The aggregated fuzzy importance weight of criterion \( C_n \), denoted as \( \tilde{W}_n = (a_n, b_n, c_n) \), assessed by \( t \) DMs is defined:

\[
\tilde{W}_n = \sum_{k=1}^{t}(l_k \otimes \tilde{W}_{nk})/\sum_{k=1}^{t} l_k, n=1,2,\ldots,m. \tag{4}
\]

The defuzzification of \( \tilde{W}_n \), denoted as \( d(\tilde{W}_n) \), can be obtained by Eq. (1). The crisp value of normalized weight for sub-criterion \( C_n \), denoted as \( W_n \), is given by

\[
W_n = \frac{d(\tilde{W}_n)}{\sum_{n=1}^{m} d(\tilde{W}_n)}, \ n = 1,2,\ldots,m, \tag{5}
\]

where \( \sum_{n=1}^{m} W_n = 1. \)

Step 3: Each decision maker defines the rate of options with respect to criteria and sub-criteria.

Assume that there is \( l \) numbers of supplier, \( A_i, i = 1,2,\ldots,l \). Let \( \tilde{R}_{inyk} = (a_{inyk}, b_{inyk}, c_{inyk}) \), be the linguistic rating of each supplier \( A_i \) on sub-criteria \( C_{ny} \), by DM \( D_k \). The
aggregated fuzzy rating of supplier $A_i$ on sub-criteria $C_{ny}$, denoted as $\bar{R}_{iny}=(a_{iny}, b_{iny}, c_{iny})$, assessed by $t$ DMs is defined as:

$$\bar{R}_{iny} = \sum_{k=1}^{t} (l_{ky} \otimes \tilde{R}_{inyk}) / \sum_{k=1}^{t} l_{ky}, n=1,2,\ldots,m, y=1,2,\ldots,z.$$  \hspace{1cm} (6)

The total rate of supplier $A_i$ on criterion $C_n$ is given by

$$\tilde{R}_{in} = \sum_{y=1}^{z} (W_{ny} \otimes \bar{R}_{iny}) , i=1,2,\ldots,l, n=1,2,\ldots,m.$$  \hspace{1cm} (7)

The subtotal fuzzy rating of each supplier on all criteria is given by

$$\tilde{s}_i = \sum_{n=1}^{m} W_n \otimes \tilde{R}_{in}, i=1,2,\ldots,l.$$  \hspace{1cm} (8)

The defuzzification of $\tilde{s}_i$, denoted as $S_i$, is therefore also given by Eq. (1).

Step 4: To this point, we have defined the status of all suppliers in all criteria and defuzzified them through Eq. (1). The final score of each supplier is obtained from Eq. (9) [1] [9].

$$\text{Score}(A_i) = \frac{S_i - \text{min}_{i=1,2,\ldots,m}(S_i)}{\text{max}_{i=1,2,\ldots,m}(S_i) - \text{min}_{i=1,2,\ldots,m}(S_i)}$$  \hspace{1cm} (9)

6. Numerical example

In this section we analyze the mentioned approach by a numerical example.

Suppose a factory wish to supply its raw materials from available suppliers. This company selects 3 candidates ($A_1$, $A_2$, $A_3$) among suppliers for more assessment purposes. This company wants to rate them using 4 criteria ($C_1$, $C_2$, $C_3$, $C_4$) and 6 sub-criteria ($C_{11}$, $C_{21}$, $C_{22}$, $C_{31}$, $C_{32}$, $C_{41}$).

Step 1: This Company selected the managers of design & engineering, manufacturing, quality assurance and sales & marketing as decision makers and assigned a weight for them according to table 2.

<table>
<thead>
<tr>
<th>DMs</th>
<th>weight</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>VH</td>
</tr>
</tbody>
</table>

Step 2: the decision makers investigated the weight of criteria and sub-criteria through Eqs. (2)- (5). Total weights were calculated in the forms of fuzzy numbers, defuzzified numbers and normalized numbers. Tables 3 and 4 shows the results.

<table>
<thead>
<tr>
<th>Sub-criteria</th>
<th>DMs' linguistic weights</th>
<th>Aggregated weights fuzzy</th>
<th>Defuzzified</th>
<th>Normalized</th>
</tr>
</thead>
<tbody>
<tr>
<td>VH</td>
<td>VH</td>
<td>VH</td>
<td>(9,10,10)</td>
<td>9.75</td>
</tr>
<tr>
<td>VH</td>
<td>VH</td>
<td>H</td>
<td>(8.6,9.8,10)</td>
<td>9.55</td>
</tr>
</tbody>
</table>

Table 2. The linguistic importance weighs of the DMs.
Table 4. The linguistic and aggregated importance weights of the criteria.

<table>
<thead>
<tr>
<th>criteria</th>
<th>DMs' linguistic weights</th>
<th>Aggregated weights</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>fuzzy</td>
<td>Defuzzified</td>
</tr>
<tr>
<td>VH VH H MH</td>
<td>(7.6,9.1,9.7)</td>
<td>8.8</td>
</tr>
<tr>
<td>H VH VH H</td>
<td>(7.8,6.9,4.6,10)</td>
<td>9.2</td>
</tr>
<tr>
<td>ML VH VH H</td>
<td>(5.9,7.7,8.7)</td>
<td>7.5</td>
</tr>
<tr>
<td>ML ML H VH</td>
<td>(4.8,6.7,4)</td>
<td>6.1</td>
</tr>
</tbody>
</table>

Step 3: the decision makers assessed the rating of suppliers through linguistic variables with respect to the sub-criteria. Total rate of each supplier on sub-criterion was calculated by Eq. (6). For instance table 5 shows the calculations of 3 sub-criteria.

Table 5. The linguistic and aggregated fuzzy ratings of sub-criteria.

<table>
<thead>
<tr>
<th>Sub-criteria</th>
<th>Suppliers</th>
<th>DMs' linguistic rating</th>
<th>Aggregated fuzzy rating</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G MG MG G</td>
<td>(61.43,80.86,95.13)</td>
<td>(40,59.7,79.74)</td>
<td></td>
</tr>
<tr>
<td>MG F MG F</td>
<td>(48.9,69.5,72.35)</td>
<td>(58.2,77.35,90.83)</td>
<td></td>
</tr>
<tr>
<td>G G MG MG</td>
<td>(61.6,82.1,96.2)</td>
<td>(58.6,79.1,94.9)</td>
<td></td>
</tr>
<tr>
<td>MG G F MG</td>
<td>(61.43,80.9,95.13)</td>
<td>(51.43,70.6,87.44)</td>
<td></td>
</tr>
<tr>
<td>G F MG MG</td>
<td>(63.6,84.3,97.44)</td>
<td>(51.43,70.6,87.44)</td>
<td></td>
</tr>
</tbody>
</table>

Table 6 shows the rating of each supplier on all criteria which have been obtained by Eq. (7). The subtotal fuzzy rating of each supplier on all criteria was calculated by Eq.(8) and the corresponded defuzzified numbers obtained through Eq. (1).

Table 6. The aggregated fuzzy and defuzzified (crisp) ratings of criteria for each supplier.

<table>
<thead>
<tr>
<th>criteria</th>
<th>Fuzzy and crisp ratings of suppliers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fuzzy</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>(61.43,80.86,95.13)</td>
<td>(40,59.7,79.74)</td>
</tr>
<tr>
<td>(48.9,69.5,72.35)</td>
<td>(58.2,77.35,90.83)</td>
</tr>
<tr>
<td>(61.6,82.1,96.2)</td>
<td>(58.6,79.1,94.9)</td>
</tr>
<tr>
<td>(61.43,80.9,95.13)</td>
<td>(51.43,70.6,87.44)</td>
</tr>
<tr>
<td>aggregated</td>
<td>(60.19,81.01,92.38)</td>
</tr>
</tbody>
</table>

Step 4: in this stage, the score of each supplier and finally the rank of them were calculated by Eq. (9).
Table 7. The final rate, score and rank of suppliers.

<table>
<thead>
<tr>
<th>Suppliers</th>
<th>Score($\sum_{g}^{21}$)</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>78.65</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>67.14</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>77.66</td>
<td>0.9</td>
<td>2</td>
</tr>
</tbody>
</table>

As you can see in this table, $A_1$ was selected as the top supplier.

7. Conclusion

In this paper we developed a multi-criteria decision making approach based on Fuzzy sets for selecting the best supplier. The reason of our work is this fact that if we select the right supplier, we can significantly save costs of purchasing processes. Moreover, the competitive advantage of the company would be improved too. So, making decisions in purchase field, which supplier selection is the most important one, is very important. When it is not possible to express performance values in numerical formats, using linguistic variables would be very effective and helpful. Therefore, in this paper we employed linguistic variables to give importance to the weights of criteria and the rates of suppliers. We also tried to coordinate our attempts about obtaining an optimized result in this field with organizational objects through assigning weights to each decision maker and also by contribution of all levels of the organization in assessment process.

Appendix A. Fuzzy operations for TFNs

If we want to use fuzzy sets in applications, we will have to deal with fuzzy numbers operations. Let $\widetilde{M} = (l, m, n)$, $\widetilde{N} = (p, q, s)$, denote triangular fuzzy numbers. Then [9] [23]:

(i) Addition of two fuzzy numbers $\oplus$

$$\widetilde{M} \oplus \widetilde{N} = (l+p, m+q, n+s)$$

(ii) Multiplication of two fuzzy numbers $\otimes$

$$\widetilde{M} \otimes \widetilde{N} = (lp, mq, ns)$$

(iii) Division of two fuzzy numbers $\Delta$

$$\widetilde{M} \Delta \widetilde{N} = (l/p, m/q, n/s)$$

(iii) Multiplication of any real number $k$ and a fuzzy number

$$k \otimes \widetilde{M} = (kl, km, kn) , \quad k > 0, k \in R.$$
REFERENCES:


